A Default Probability Estimation Model: 
An Application to Japanese Companies

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Abstract

On the assumption that asset value of a company is the sum of the total amount of current price of stock and debt value, estimation was made with the first moment and second moment concerning a mean value and variance of the sum. We also assume a new variable for which fluctuation during an evaluation period conforms to these moments and follow geometric Brownian motion. Then we construct a default probability estimation model on condition that the variable is regarded as the asset value of the company. For constructing expected default probability (EDP) model, we partially follow Levy’s(1992) way in which a new variable to be used for average option is assumed. Thus its evaluation formula is derived. In addition, concerning estimated values of the default probability the model introduced this time is compared with the conventional structural approach with respect to the company in Japan where default was actually caused and the company free from the default.

Keywords. Expected default probability (EDP), Moment, Black-Scholes-Merton model

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1. **Introduction**

Since 1980 international monetary market has been expanded with great strides. On the other hand, severity of cumulative debts has been deepened and risk control of increasing derivative transactions has become a problematic point. For this reason, it becomes very important not only for financial institutions but also for general companies in possession of stocks or bonds to evaluate management abilities, credit power, etc. of companies in an fair and simple manner. Especially with the financial institutions including banks, soundness in management is required in performing international business endeavors and in making transactions competitive enough to cope with foreign financial institutions in a field of international competitions. Thus, econometrical measures of credit risks have become indispensable in completing satisfaction of BIS restriction (Basel Agreement), an international unification criterion determined by Basel Bank Supervising Committee. In controlling such a kind of credit risks, an estimation method of the default probability for the clients with whom credit should be provided becomes important. Proper estimation of the default probability becomes very important information when observation is made with proper pricing of the bond values to which credit risks are applied or with adequate profit brought about by market transactions, resulting in foundation to evaluate these factors.

Modeling of the credit risks are roughly classified into three types, (1) a traditional method where expected loss values are estimated and evaluated dependent on the estimated values obtained from the past data, (2) structural approach where an event with which a company loses solvency to be defaulted on the given balance sheet is modeled to obtain a default probability of the company, and (3) inductive approach where the default probability is exogenously given in a probability model.

As a representative method of the structural approach taken up in this study, Merton [1974], who stipulates as default a situation where asset values become below liabilities as default, models default risks of the bonds by using an option pricing theory introduced by Black and Scholes[1973]. The approach concerning the evaluation of the company debt of Black and Scholes[1973] and Merton[1974] has become an indispensable portion of the corporate finance theory. After that, Black and Cox[1976] developed a model that default is caused at the time when the value of the profit of the company reaches a certain low threshold value by relaxing the assumption in question taking up a framework of the
Merton model as a foundation. On the other hand, Longstaff and Schwarz [1995] expands the Black and Cox model judging from the mechanism explaining that the company is fallen into the default state when the company value following a stochastic process narrowly fails to reach a threshold value by incorporating a probability fluctuation model of an interest rate for the purpose of uncertainty of the interest. At that time with the Black-Scholes-Merton model as the conventional structural approach, the EDP estimation value is occasionally underestimated for heightening of volatility usually caused by critical affairs of the company (e.g., in the vicinity of the default time point) and increase of a ratio of the leverage (e.g., in lowering of total amount of current price of stock).

In this study, the EDP estimation model is constructed by assuming a new variable which follows geometric Brownian motion allowing the fluctuation during the EDP evaluation period conforms to the first moment and second moment of the sum of the total amount of current price of stock and debt value with the variable being regarded as asset value of the company. By so doing, we propose a model that gives more influence to EDP estimates than the traditional model for volatility of the total amount of current price of stock susceptible to influence of trend of the market and a ratio of the leverage showing financial state of the company. Then, the proposed model is compared with the conventional model, showing difference of the results of default and non-default companies in Japan.

2. Black-Scholes-Merton model

Default risks are modeled by using an option pricing theory brought about by the Black-Scholes-Merton model in consideration of a simple company comprised of a single type of the debt that is free from profit payment and a single type of capital that is liberated from dividend on the assumption that a situation where asset value becomes below the debt is default. Then the assumption below is made with the EDP estimation by means of the Black-Scholes-Merton model.

We assume that the asset value $A(t)$ of the company follows a stochastic process below

$$dA(t) = \mu_A A(t)dt + \sigma_A A(t)dW(t) \quad (1)$$

where $\mu_A$ is the expected profit ratio of the asset value, $\sigma_A$ is volatility of the asset value.
value and \( dW(t) \) is standard Brownian motion. From (1) the asset value \( A(\tau) \) of the time point can be expressed as shown below for the asset value \( A(T) \).

\[
A(\tau) = A(T) \exp \left( \left( \mu_A - \frac{\sigma^2_A}{2} \right) \tau + \sigma_A W(\tau) \right),
\]

(2)

where \( W(\tau) \sim N(0, \tau) \). Since, log value of \( A(\tau) \) is expressed as

\[
\ln A(\tau) = \ln A(T) + \left( \mu_A - \frac{\sigma^2_A}{2} \right) \tau + \sigma_A W(\tau),
\]

(3)

\( \ln A(\tau) \) is distributed as a normal distribution with the mean \( \ln A(T) + \left( \mu_A - \frac{\sigma^2_A}{2} \right) \tau \), variance \( \sigma^2_A \tau \). Based on the assumption shown above, let the default be defined as an event where the asset value becomes below the debt value at the time point \( T \). Denoting the debt value at the time point \( T \) by \( B(T) \), the default event can be expressed as \( A(T) \leq B(T) \). The default probability EDP on the Black-Scholes-Merton model is given as shown below.

\[
EDP = P(\ln A(T) \leq \ln B(T))
= P \left\{ \ln \left( \frac{A(T)}{A(T_0)} \right) - \frac{\left( \mu_A - \frac{\sigma^2_A}{2} \right) (T - T_0)}{\sigma_A \sqrt{T - T_0}} \leq \frac{\ln(\frac{B(T)}{A(T_0)}) - \left( \mu_A - \frac{\sigma^2_A}{2} \right) (T - T_0)}{\sigma_A \sqrt{T - T_0}} \right\},
\]

Letting

\[
Z = \frac{\ln \left( \frac{A(T)}{A(T_0)} \right) - \left( \mu_A - \frac{\sigma^2_A}{2} \right) (T - T_0)}{\sigma_A \sqrt{T - T_0}},
\]

the probability above becomes

\[
1 - P \left( Z \geq \frac{\ln \left( \frac{B(T)}{A(T_0)} \right) - \left( \mu_A - \frac{\sigma^2_A}{2} \right) (T - T_0)}{\sigma_A \sqrt{T - T_0}} \right) = 1 - N \left( \frac{\ln \left( \frac{A(T_0)}{B(T)} \right) + \left( \mu_A - \frac{\sigma^2_A}{2} \right) (T - T_0)}{\sigma_A \sqrt{T - T_0}} \right),
\]

(4)

where \( N(\cdot) \) is an accumulated probability density function of the standard normal distribution.
3. Parameters required for estimation of the default probability on the Black-Scholes-Merton model

With estimation of the default probability based on (4), it is necessary to set values of 4 parameters $A(T_0)$, $B(T)$, $\mu_A$, and $\sigma_A$ except for $T-T_0$ given exogenously. In the empirical study EDP estimation is made by Mercus and Shaked.[1984], Ronn and Verma.[1989], Saito and Morihira[1997] and Ando and Marumo[2001] with $A(T_0)$, $B(T)$, and risk-free interest rate, whereas the estimation is made by Miyoshi[1998] with $A(T_0)$, $\sigma_A$, and risk-free interest rate giving exogenously. Meanwhile although risk-free interest rate is used taking up risk neutrality as premise with these types of EDP estimate, Morihira [1997] proposed that EDP should be estimated by utilizing the Boness[1964] model as an option evaluation model with which none of risk neutral evaluation is made. In this study, assumption is made as shown below for the 4 parameters.

a) Asset value $A(T_0)$ at the time point $T_0$.

Let the asset value be the sum of the total amount of current price of stock $E(T_0)$ and debt value $B(T_0)$. That is to say, the asset value can be expressed as shown below on the condition that the stock value $S(T_0)$ and issued stock number $n$ are known.

$$A(T_0) = E(T_0) + B(T_0) = nS(T_0) + B(T_0)$$

(5)

b) Debt value $B(T)$ on the time point $T$.

Debt value cannot be observed in the market, and therefore let it be understood that $B(T) = B(T_0) = “booking value of the closest debt.”$

c) Expected profit ratio $\mu_A$ of the asset value.

Let $\mu_A = r$ with the risk-free interest rate $r$.

d) Volatility $\sigma_A$ of the asset value.

On the assumption that the asset value is the sum of the total amount of current price of stock and debt value, the expression shown below is obtained from Itô’s lemma because the volatility $\sigma_E$ is observable on the market,

$$\sigma_E E(T_0) = \frac{\partial E(T_0)}{\partial A} \sigma_A A(T_0)$$

(6)
where

\[ \sigma_E E(T_0) = N \left( \frac{\ln(A(T_0)/B(T_0)) + (\mu_A + \sigma_A^2/2)(T - T_0)}{\sigma_A \sqrt{T - T_0}} \right) \sigma_A A(T_0). \] (7)

Then \( \sigma_A \) satisfying (7) is found. With volatility \( \sigma_E \) of the total amount of current price of stock, historical volatility calculated from stock data in the past is used.

Note: It is easy to see that as \( T \to T_0 \), \( \sigma_A \to \frac{E(T_0)}{B(T_0) + E(T_0)} \sigma_E \).

4. Estimation of the first and second moments of the sum of the total amount of current price stock and debt value and derivation of an evaluation formula

As stated in the previous section, the asset value of the company is supposed as the sum of the total amount of current price of stock and debt value. For constructing EDP model, we partially follow Levy's(1992) way in which a new variable to be used for average option is assumed.

a) The first and second moments such as a mean value and variance of the sum of the total amount of current price of stock and debt value together are derived.

b) A new variable \( X \) following geometric Brownian motion where fluctuation in the EDP evaluation period coincides with the moments of the sum of the stock price current value and debt value be assumed.

c) By regarding the variable as asset value of the company, EDP estimation is made.

On the assumption that asset value \( A(T) \) of the \( T \) th period is the sum of the total amount of current price of stock and debt value, the asset value can be expressed as shown below.

\[ A(T) = E(T) + B(T). \] (8)

Let the evaluation time point be \( t \) and the debt value \( B(T) \) be a value of \( B(T) \equiv B(T_0) \). The first and second moments \( E[A(T)] \) and \( E[A(T)^2] \) of \( A(T) \) can be expressed as shown below.
\[ E[A(T)] = E[E(T) + B(T_0)] \quad \text{and} \quad E[A(T)^2] = E[(E(T) + B(T_0))^2] \quad (9) \]

On the assumption that total amount of current price of stock \( E(T) \) follows geometric Brownian motion, the total amount of current price of stock \( E(\tau) \) on the \( \tau(T_0 \leq \tau \leq T) \) time point can be expressed as shown below with the total amount of current price of stock \( E(T_0) \) of the \( T_0 \) period.

\[ E(\tau) = E(T_0) \exp\left(\left(\mu_E - \frac{\sigma_E^2}{2}\right)\tau + \sigma_E W(\tau)\right) \quad (10) \]

From the above description, the first and second moments of the sum of the total amount of current price of stock and debt value are expressed as shown below.

\[
\begin{align*}
E[A(T)] &= E(T_0) e^{\mu_\tau (T-T_0)} + B(T_0), \\
E[A(T)^2] &= E(T_0)^2 e^{(2\mu_\tau + \sigma_\tau^2)(T-T_0)} + 2E(T_0) B(T_0) e^{\mu_\tau (T-T_0)} + B(T_0)^2. \quad (11)
\end{align*}
\]

At the next stage, define a new variable \( X \) which follows the geometric Brownian motion. We also assume that the variable \( X(t) \) follows the stochastic process shown below.

\[ dX(t) = \mu_x X(t) dt + \sigma_x X(t) dW(t). \quad (12) \]

From (12), it is explained that the variable \( X(\tau) \) on the \( \tau(T_0 \leq \tau \leq T) \) time point is expressed as shown below with the variable \( X(T_0) \) on the \( T_0 \) time point.

\[ X(\tau) = X(T_0) \exp\left(\left(\mu_x - \frac{\sigma_x^2}{2}\right)\tau + \sigma_x W(\tau)\right) \quad (13) \]

From the above, the first moment \( E[X(T)] \) and second moment \( E[X(T)^2] \) of the variable \( X \) are expressed as shown below.

\[
\begin{align*}
E[X(T)] &= X(T_0) e^{\mu_x (T-T_0)}, \\
E[X(T)^2] &= X(T_0)^2 e^{(2\mu_x + \sigma_x^2)(T-T_0)} \quad (14)
\end{align*}
\]

In this study, since we assume that the fluctuation during the evaluation period of the variable coincides with the first and second moments of the sum of the total amount of current price of stock and debt value drift ratio \( \mu_x \) and volatility \( \sigma_x \) can be from (11) and (14) obtained as follows.

\[ X(T_0) e^{\mu_x (T-T_0)} = E(T_0) e^{\mu_\tau (T-T_0)} + B(T_0) \]

Hence,
\[ \mu_x = \frac{1}{T - T_0} \ln \left( \frac{E(T_0)}{X(T_0)} e^{\mu_x(T - T_0)} + \frac{B(T_0)}{X(T_0)} \right) \]

and for \( \sigma_x \)

\[ X(T_0)^2 e^{(2\mu_x + \sigma_x^2)(T - T_0)} = E(T_0)^2 e^{(2\mu_x + \sigma_x^2)(T - T_0)} + 2E(T_0)B(T_0)e^{\mu_x(T - T_0)} + B(T_0)^2 \]

gives

\[ \sigma_x = \frac{1}{T - T_0} \ln \psi - 2\mu_x \]

where

\[ \psi = \frac{E(T_0)^2}{X(T_0)^2} e^{(2\mu_x + \sigma_x^2)(T - T_0)} + 2E(T_0)B(T_0)e^{\mu_x(T - T_0)} + \frac{B(T_0)^2}{X(T_0)^2} \]

**Note:** As \( T \to T_0 \) \( \sigma_x \to \frac{E(T_0)}{B(T_0)\sqrt{T - T_0}} \sigma_E \).

On the assumption that the company concerned has not been liquidated till time \( T \) the total amount of current price of stock \( \tilde{E}(T_0) \) at time \( T_0 \) is expressed as the difference between the asset value and debt value. In other words,

\[ \tilde{E}(T_0) = X_0 e^{(\mu - \sigma^2/2)(T - T_0)} N(d) - B(0) e^{-r(T - T_0)} N(d - \sigma \sqrt{T - T_0}) \] (15)

with

\[ d = \frac{\ln(X(T_0)/B(0)) + (\mu_x + \sigma_x^2/2)(T - T_0)}{\sigma_x \sqrt{T - T_0}} \]

\[ \mu_x = \frac{1}{T - T_0} \ln \left( \frac{E(T_0)}{X(T_0)} e^{\mu_x(T - T_0)} + \frac{B(T_0)}{X(T_0)} \right) \]

\[ \sigma_x = \frac{1}{\sqrt{T - T_0}} \ln \psi - 2\mu_x \]

\[ \psi = \frac{E(T_0)^2}{X(T_0)^2} e^{(2\mu_x + \sigma_x^2)(T - T_0)} + 2E(T_0)B(T_0)e^{\mu_x(T - T_0)} + \frac{B(T_0)^2}{X(T_0)^2} \]

This is equivalent to a payoff of European call option by letting the underlying asset be the asset value of the company concerned and the exercise price be the debt value.

From the above description, let the default be defined as an event where the variable \( X \)
becomes below debt value on the time point $T$. With the default event, the default probability $EDP$ on the time point $T$ viewed from the evaluation time point $T_0$ of the variable is given as shown below when $X(T) \leq B(T_0)$.

$$EDP = P(\ln X(T) \leq \ln B(T_0)) = 1 - N\left(\frac{\ln(X(T_0)/B(T_0)) + (\mu_e - \sigma^2 x / 2)(T-T_0)}{\sigma x \sqrt{T-T_0}}\right)$$  \hspace{1cm} (16)

We note that $EDP$ values don't change even by any value of $X(T_0)$ ($0 < X(T_0) < \infty$) at evaluation time. Note that all the parameters in (16) necessary to estimate default probability are those as explained in section 3. Also, it is understood that $\mu_e = r$ and the risk-free interest rate is equivalent to $\mu_A = r$.

5. Empirical analysis of the default company

In this section, a relation between the model proposed in this paper and the estimation method of the default probability by the Black-Scholes-Merton model is examined. Default probabilities are calculated with an company in Japan in which default was actually caused and another company of the same trade in which no default was caused. After that, comparison is made with both the default probabilities. Details of the companies in which the default probability is compared and data used in the company are listed in Tables 1 and 2. Financial indices of the individual companies, which are extracted from NEEDS-Financial QUEST, are utilized. Meanwhile with the risk-free interest rate, the Japanese bond interest rate on an annual basis calculated by Bloomberg Corporation was used.

5.1 Transition of the EDP estimation value of both the models

5.1.1 A default company and non-default company in the electric appliance manufacturer

EDP transition of the proposed model and conventional model in the electric appliance manufacturer is illustrated in Figure 1. With Akai Denki as a default company and TEAC
as a non-default company, comparison is made between the model proposed and the Black-Scholes-Merton model. From this it is observed that transition of the EDP shows the same tendency with the individual models. However, the EDP estimation value of the model of this time is always kept in a high state. The tendency, which is strong enough when the EDP is high, is so strong especially in the vicinity of the default time point.

With the conventional model, the transition of Akai Denki as a default company, which becomes 42.1% on August 1, 2000 before default, is once decreased to recover default. The EDP estimation value on the default time point (September 2, 2000), which is at that time 30.1% , remains 47.9% since that time on as well. As seen above, the transition, which is very low also with respect to the EDP since the default time point, does not show adequately the state in the vicinity of the default time point. Contrary to the above, the proposed model, which showed 59.5% on August 1, 2000, is once lowered to reach on the time point where default was made, anew rises to 99.8% since that time on. As seen above for example, the state of default is adequately exhibited. In such a manner, a considerable amount of dissociation is seen with the estimation value of the EDP in the vicinity of the default time point.

When transition of the EDP of both the companies is compared, a tendency time-series-dependently similar at the primary period is observed, and the degree of the transition is very small. Especially in the vicinity of the default time point of Akai Denki, the EDP of TEAC is exceedingly small. After that, the EDP of TEAC repeats fluctuation of heightening and lowering. These kinds of transition can be recognized with both the models.

5.1.2 Transition of the EDP of the default company and non-default company in retail business

In connection with Mical and Aeon as retail dealers, the same tendency is exhibited with the EDP transition of the proposed model and conventional model as illustrated in Figure 2, and the EDP estimation value of the model of this time is kept in a very high state. Meanwhile in case of the transition of Mical as a default company, a considerable amount of dissociation is seen with the EDP of both the models in the vicinity where default was made (September 14, 2001). In the meantime, the default point comes up to 26.8% with the proposed model, and this becomes 94.1% in October 2001 to be increased to 98.0% in
December 2001. Contrary to the above, the EDP of the default point reaches 22.7% in case of the conventional model. On October 2001, the EDP becomes 39.2% to be decreased to 21.7% in December 2001. Thus contradictory behavior is noticed as seen for example in decrease of the EDP after the default.

When the transition of the EDP of both the companies to the extent of the default time point of Mical is compared, the degree of the transition is, although a time-series-dependently similar tendency is observed, as small as in the case of electric appliance manufacturers. Especially in the vicinity of the default time point of Mical, the EDP of Aeon is exceedingly small and always shows stabilized low value after that. With these, the same tendency is noticed for both the models.

5.2 Relation among the EDP, leverage, and volatility of the stock current price total amount of both the models

5.2.1 Relation between the EDP and leverage of both the models

A relation between the EDP and the leverage (B/A) with both the models with respect to the electric appliance manufacturer and retail sale is illustrated, respectively, in Figures 3 and 4. In the electric appliance manufacturer, defaulted Akai Denki makes great change with the value of B/A in a range from 0.32 to 0.99 in comparison with TEAC. There is a tendency that the EDP is increased accompanied with increase of B/A, but variance is large enough even for the same B/A. Thus it cannot be said that their correlation is distinct.

In connection with the EDP of Akai Denki in Figure 3(a), the proposed model shows 99.8% when B/A = 0.99. Contrary to the above, the model shows 47.9% in case of the conventional model even when B/A = 0.99. Thus it can safely be said that great difference exists.

With TEAC as a non-default company, the value of B/A is distributed in a range from 0.48 to 0.85. Meanwhile in comparison with Akai Denki, variance of the EDP for B/A is small. From the spot where B/A comes to exceed 0.7, there appears to be a tendency that the EDP will partially be increased. However none of exceeding dissociation between the proposed model and the conventional model can be seen.

With Mical as a retail dealer shown in Figure 4(a), the EDP is rapidly increased on the
time spot where it approaches B/A = 1.0, so that difference between both the models is extremely great. Aeon as a non-default company exhibits a value low enough to be B/A <0.56, whereas 5.04% is the highest value with the proposed model even in relation to the EDP. Thus none of great difference can be noticed with both the models.

5.2.2 Relation between the EDP of both the models and the volatility of the stock current price total amount

In Figures 5 and 6, a relation between the EDP and the $\sigma_E$ is illustrated with respect to the electric appliance manufacturer and retail sale. The EDP and $\sigma_E$ of Akai Denki as a default company of the electric appliance manufacturer apparently show a positive correlation, and there is a tendency likewise with both the models that the EDP will be increased with the increase of the $\sigma_E$. From this it is understood that the EDP is greatly subjected to influence of the $\sigma_E$.

In comparison with both the models, it is revealed that an increasing ratio of the EDP to the $\sigma_E$ of the proposed model is large. Furthermore it is also revealed that as the $\sigma_E$ becomes large, difference of the EDP between both the models is made increased. It is understood that especially with $\sigma_E > 6.43$ in the vicinity of the default time point of Akai Denki, difference is exceedingly increased. In a region where the $\sigma_E$ is small, dependence of the EDP on the $\sigma_E$ shows the same tendency with those of Akai Denki and TEAC.

With Mical as a default company of retail sale, a tendency the same as that of Akai Denki is noticed as illustrated in Figure 6(a). With the model of this time, the EDP exceeds 94.1% at $\sigma_E > 4.85$, approaching 100% with increase of the $\sigma_E$. Contrary to the above, the EDP is reduced to 21.7% for increase of the $\sigma_E$ after the maximum EDP = 40.1% was shown at $\sigma_E = 5.19$ with the conventional model.

With Aeon as a non-default company, the $\sigma_E$ is as small as 0.74 at the greatest. However, there is a tendency that the EDP will increase with increment of the $\sigma_E$ with respect to both the models.
5.3 Difference of the EDP between both the models

The EDP obtained with the model proposed always shows higher values than those of the conventional model. To explain this matter, a relation between the difference of the EDP obtained with the both models and the B/A together with $\sigma_k$ was examined.

5.3.1 Relation between the difference between the EDP of both the models and the leverage

Judging from difference of the EDP of the electric appliance manufacturer and the B/A as illustrated in Figure 7(a), it is understood that values of the B/A are distributed in a range from 0.32 to 0.99 with both the companies. On the whole, dispersion of the difference of the EDP for the B/A of Akai Denki as a default company is heightened. Influences of the difference of the EDP for B/A of Akai Denki exhibit similarity in the vicinity of the region ranging from 0.32 to 0.76 with both the companies. However after the B/A reaches 0.8, difference of the EDP of Akai Denki is heightened. When the value of the difference comes to approach 1, difference of the EDP becomes more than 51.9% allowing the EDP estimation value of both the model to be extremely dissociated.

Although difference of the EDP is small for the B/A <0.98 with both the companies with respect to the difference of the EDP of retail sale as illustrated in Figure 7(b), difference of the EDP of Mical becomes more than 54.8% when the B/A approaches 1. Thus the EDP estimation value of both the models is made exceedingly dissociated.

5.3.2 Relation between the difference of the EDP of both the models and the volatility of the total amount of current price of stock

With Akai Denki and TEAC as electric appliance manufacturers, a relation between the EDP of both the models and the volatility of the stock price current total amount is illustrated in Figure 8(a). The relation of the difference of the EDP for $\sigma_k$ is showed to be almost on the same curve with both the default company and the non-default company. Furthermore especially with increase of the $\sigma_k$, there is a tendency that difference of the EDP will be increased.
With the $\sigma_E$ of the total amount of current price of stock of both the companies, distribution is showed in a range from 0.35 to 6.43 in case of Akai Denki, while distribution of TEAC is showed in a range from 0.24 to 1.08. Difference of the EDP for the $\sigma_E$ is made greater uniformly as the $\sigma_E$ of both the companies become increased. In such a situation, difference of the EDP is made greater as the $\sigma_E$ of Akai Denki as a default company is increased. On a spot where the $\sigma_E$ exceeds 6.11, difference of the EDP remains as high as 52.0%. This corresponds to the fact that in the vicinity of the default time point, difference of the EDP estimation value between the proposed model and the conventional model has become greater.

With Mical and Aeon as retail dealers, a relation between the EDP of both the models and the volatility of the total amount of current price of stock is illustrated in Figure 8(b). With influence of the difference of the EDP for the $\sigma_E$, difference of the EDP for the $\sigma_E$ is made greater as the $\sigma_E$ of Mical as a default company is increased as seen in the result of the electric appliances manufacturer referred to before. In such a situation, difference of the EDP is made greater as the $\sigma_E$ of Mical is increased. When the $\sigma_E$ exceeds 4.85, difference of the EDP becomes more than 54.8 %. This corresponds to the fact that in the vicinity of the default time point likewise in case of Akai Denki, difference of the EDP estimation value between the proposed model and the conventional model has become greater.

The reason why the EDP becomes greater in the vicinity of the default time point with the proposed model corresponds to the fact that the $\sigma_E$ becomes larger. From these results, it is understood that differences for the $\sigma_E$ and for the EDP are shown by almost the same relation regardless of how types of business are or irrespective of whether the company is of default or non-default.

With the conventional model, influence of the volatility of the asset value of the companies for the $\sigma_E$ is, as seen above, small even if the $\sigma_E$ susceptible to influence of the trend of the market is high enough when the total amount of current price of stock approaches 0. Therefore in the vicinity of the default time point for example, the value of the EDP has been underestimated. By considering the moment of the sum of the stock
current total amount and debt value as movement of the asset value of the companies, it seems likely that the $\sigma_E$ rather than the conventional model corresponds to the EDP, and default situation can be shown distinctly by grasping trend of the market.

6. Conclusions

On the assumption that the asset value of a company is the sum of the total amount of current price of stock and debt value, first and second moments such as a mean value or variance of the sum were estimated. We also assumed that a new variable for which fluctuation during the EDP evaluation period conforms to the moments and follow geometric Brownian motion. After that, an EDP estimation model was proposed by regarding the variable as asset value of the company. Thus an evaluation formula of the model was introduced.

By the model of this time, transition of the EDP of 2 types of business of default company and non-default company was obtained. After that, comparison was made with the Black-Sholes-Merton model. The same tendency such as time-series-dependent change of the EDP of both the models, difference between companies, etc. is shown. However on all the time points, the EDP of the proposed model showed a higher value than that of the conventional model. The tendency is so strong when the EDP is high enough, and movement differing from that of the conventional model is observed especially in the vicinity of the default time point.

For heightening of volatility of the total amount of current price of stock in the vicinity of the default time point of the company and increase of the ratio of the leverage, the EDP estimation value is occasionally underestimated with the conventional model. However, it is possible for the proposed model, where the EDP estimation value in the default time point of the company shows almost 100 %, to show an actual default situation more distinctly than the conventional model can. This stems from the reason that owing to the volume of the influence of the volatility of the total amount of current price of stock to be exercised on the difference of the EDP of both the models, the parameter of the $\sigma_E$ liable to be more strongly influenced by the trend of the market exercises greater influence on the EDP estimation value than the conventional model does.
### Table 1 Default and Non-Default Company

<table>
<thead>
<tr>
<th>A type of business</th>
<th>Default enterprises</th>
<th>Default time point</th>
<th>Non-default enterprises</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric appliance</td>
<td>Akai Denki</td>
<td>2000/11/2</td>
<td>TEAC</td>
</tr>
<tr>
<td>manufacturer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retail business</td>
<td>Mical</td>
<td>2001/9/14</td>
<td>Aeon</td>
</tr>
</tbody>
</table>

### Table 2 Available Data

<table>
<thead>
<tr>
<th>Financial statements</th>
<th>Separate closing accounts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free interest rate</td>
<td>One year Japanese bond interest rate</td>
</tr>
<tr>
<td>Volatility of the total amount of current price of stock</td>
<td>Annual conversion of the logarithm earning rate for 60 days of a stock price</td>
</tr>
<tr>
<td>Evaluation period</td>
<td>One year</td>
</tr>
</tbody>
</table>
Figure 1: EDP values for Akai Denki and TEAC

(a) Akai Denki

(b) TEAC

Figure 2: EDP values for Mical and Aeon

(a) Mical

(b) Aeon
Figure 3: Relation between the EDP values and the leverage for electric appliance manufacturers

Figure 4: Relation between the EDP values and the leverage for retail business
Figure 5: Relation between the EDP values and $\sigma_E$ for electric appliance manufacturers

Figure 6: Relation between the EDP values and $\sigma_E$ for retail business
Figure 7: Relation between the difference of EDP values and the leverage

Figure 8: Relation between the difference of EDP values and $\sigma_E$
References


